June 2017

EE 272 - Dynamics of Lasers
Homework 4 : Mode-locking
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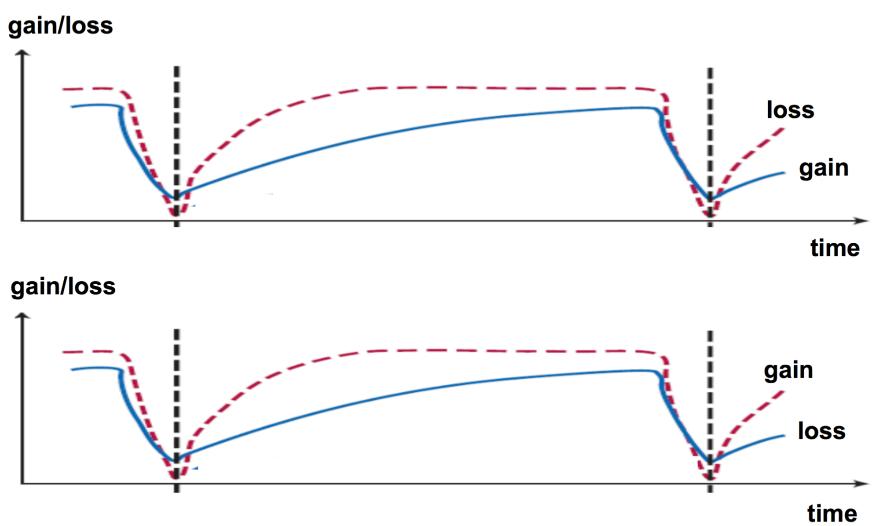
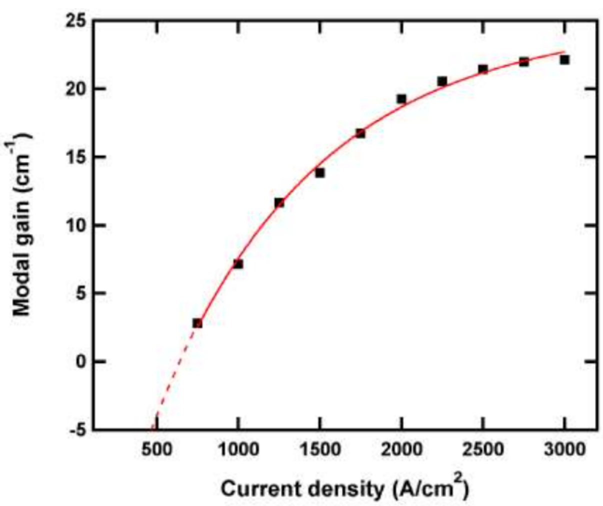
Mode-locking of two-section devices can be achieved without the need of external signals, by taking advantage of the nonlinear dynamics of the intracavity absorber. Let us consider a resonator where a reverse voltage V_r is applied onto the absorber section of the laser and current density J is injected into the gain section. For injection currents above the threshold value, an optical waveform is allowed to propagate inside the cavity, being amplified in the gain section and attenuated in the absorber. If the waveform includes a sharp temporal feature with enough energy to induce loss saturation at the absorber, the latter will become transparent during a short time interval before loss recovery takes place. This will lead to the formation of a short pulse propagating into the gain section. In what follows, a_0 is the unsaturated absorption in the absorber region, g_0 the modal gain, L_a the length of the absorber and L_g the length of the gain section.

Question 1 The figure below represents two possible gain and loss time dynamics in the laser diode. In your opinion, which one leads to mode-locking operation ? Justify your response.



The first curve leads to mode-locking. Upon start-up of laser emission, the laser modes initially oscillate with relative phases that are random such that the radiation pattern consists of noise bursts. If one of these bursts is energetic enough to provide a fluence that matches the saturation fluence of the absorber, it will bleach the absorption. This means that around the peak of the burst where the intensity is higher, the loss will be smaller, while the low-intensity wings become more attenuated. The pulse generation process is thus initiated by this family of intensity spikes that experience lower losses within the absorber carrier lifetime. The dynamics of absorption and gain play a crucial role in pulse shaping. In steady state, the unsaturated losses are higher than the gain. When the leading edge of the pulse reaches the absorber, the loss saturates more quickly than the gain, which results in a net gain window. The absorber then recovers from this state of saturation to the initial state of high loss, thus attenuating the trailing edge of the pulse. It is thus easy to understand why the saturation fluence and the recovery time of the absorber are of primary importance in the formation of mode-locked pulses. As a conclusion, ultrafast carrier dynamics are fundamental for successful mode locking in semiconductor lasers, particularly in the saturable absorber, because the absorption should saturate faster and recover faster which is a determining factor for obtaining ultrashort pulses.

Question 2 The figure below shows the modal gain g_0 versus the pump current density J at a wavelength of 1.59-μm. The black squares are the measured data while the red solid line is the curve-fitting. Estimate : 1) the differential absorption $da_0/dJ \approx dg_0/dJ|_{g_0=0}$; 2) the differential gain dg_0/dJ taking the value near to the maximum modal gain value for J= 2750 A/cm^2. Can this laser get mode-locked ?



Using a linear regression, one find $da_0/dJ \approx dg_0/dJ|_{g_0=0} \approx 0.003$ cm/A and $dg_0/dJ|_{J=2700A/cm^2} \approx 0.03$ cm/A. From the criteria seen in class, we see that $da_0/dJ >> dg_0/dJ$ meaning that the condition for mode-locking is fulfilled.

Question 3 In a recent paper published by Lin et al. (see Optics Express, Vol. 17, pp. 10739, 2009), a more complete expression of the mode-locking criteria was defined. By accounting for the distribution of the gain and absorption in separate sections of the cavity, the novel criteria is redefined such as :

$$\frac{a_0}{g_0}\frac{L_a}{L_g} > \left(\frac{\frac{dg_0}{dJ}}{\left.\frac{dg_0}{dJ}\right|_{g_0=0}} \right)^2 \tag{1}$$

In the table below (α_m is the transmission loss coefficient of each laser), we examined three different passively mode locked lasers with different cavity lengths. For instance a 4-mm cavity length device with a 0.5-mm absorber and a 3.5-mm gain section is abbreviated as $A_{0.5}G_{3.5}$. Compare the mode-locking conditions obtained with the criteria used in the previous question with those from with Eq. (1) ? Conclusions ?

Hint : Express the threshold condition in the laser cavity as a function of (g_0, a_0). Then, to determine g_0, assume an internal loss coefficient of α_i=14 cm^{-1}.

	$A_{0.5}G_{3.5}$	$A_{0.5}G_{3.0}$	$A_{0.3}G_{2.0}$
α_m (cm^{-1})	1.48	1.7	2.58
a_0 at 1.59-µm (cm^{-1})	17.5 (Vr=1V)	17.5 (Vr=1V)	18 (Vr=2V)
dg_0/dJ (cm/A)	0.0073	0.0021	0.0008
dg_0/dJ at g_0=0 (cm/A)	0.018	0.018	0.018

To use Eq. (1), the modal gain g_0 has to be calculated. In order to do so, the threshold condition is determined by taking into account the loss spreading out all along the laser cavity i.e. in both gain and absorber sections.

$$e^{g_0 L_g - a_0 L_a - (\alpha_i + \alpha_m)(L_a + L_g)} = 1 \tag{2}$$

leading to,

$$g_0 L_g = a_0 L_a + (\alpha_i + \alpha_m) L \tag{3}$$

For the three lasers described in the table, using Eq. (3) leads to g_0=20.2 cm^{-1} (case 1), 21.2 cm^{-1} (case 2), and 21.7 cm^{-1} (case 3). Then, we get :

- $A_{0.5}G_{3.5}$: $\frac{a_0}{g_0}\frac{L_a}{L_g} \approx .125 < \left(\frac{\frac{dg_0}{dJ}}{\frac{dg_0}{dJ}\Big|_{g_0=0}}\right)^2 \approx .165$ (cannot get mode-locked) ;
- $A_{0.5}G_3$: $\frac{a_0}{g_0}\frac{L_a}{L_g} \approx .137 > \left(\frac{\frac{dg_0}{dJ}}{\frac{dg_0}{dJ}\Big|_{g_0=0}}\right)^2 \approx .013$ (can get mode-locked) ;
- $A_{0.3}G_2$: $\frac{a_0}{g_0}\frac{L_a}{L_g} \approx .125 >> \left(\frac{\frac{dg_0}{dJ}}{\frac{dg_0}{dJ}\Big|_{g_0=0}}\right)^2 \approx .002$ (can easily get mode-locked).

From the criteria seen in class and the previous question, we see that all lasers can get mode-locked. However, this condition does not take the structure of the cavity into consideration. This inequality is not particularly instructive for designing two-section mode-locked lasers. Indeed, from the application of Eq. (1), it turns out that the $A_{0.5}G_{3.5}$ laser cannot get mode-locked while it does from the first condition.